

# Glauber Gluons in Soft Collinear Effective Theory and Factorization of Drell-Yan Processes

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## Abstract

Glauber gluons in Drell-Yan processes are soft gluons with the transverse momenta much larger than their momentum components along the directions of initial hadrons. Their existence has been a serious challenge in proving the factorization of Drell-Yan processes. The recently proposed soft collinear effective theory of QCD can provide a transparent way to show factorizations for a class of processes, but it does not address the effect of glauber gluons. In this letter we first confirm the existence of glauber gluons through an example. We then add glauber gluons into the effective theory and study their interaction with other particles. In the framework of the effective theory with glauber gluons we are able to show that the effects of glauber gluons in Drell-Yan processes are canceled and the factorization holds in the existence of glauber gluons. Our work completes the proof or argument of factorization of Drell-Yan process in the framework of the soft collinear effective theory.

QCD factorization theorems enable us to make theoretical predictions from QCD for hard-scattering processes, where one can separate long-distance effects from short-distance effects. The short-distance effects depend on the details of a process and can be studied with perturbative QCD, while long-distance effects can be characterized by matrix elements of QCD operators, which depend only on the structure of hadrons and infrared properties of QCD[1]. In general, it is difficult to prove a factorization theorem, e.g., much effort has been spent to prove factorization for Drell-Yan process[2, 3]. A main obstacle has been the exchange of glauber gluons in initial state interactions[4, 3]. The glauber gluons are soft with the transverse momenta much larger than their momentum components along the directions of initial hadrons.

Recently, the soft collinear effective theory(SCET) has been proposed to study interactions among collinear- and various soft particles of QCD[5, 6, 7]. The interactions are at long-distance and produce collinear- and I.R. divergences in perturbative calculations. Assuming that SCET includes all those particles of QCD, then SCET will re-produce these collinear- and I.R. singularities. It is interesting to note that with SCET the proofs or arguments of factorization theorems can be simplified or made in a transparent way. In [6] several examples including Drell-Yan processes are given. However, the glauber gluons are not included in the proposed SCET[5, 6]. In this letter we make an attempt to include the glauber gluons into SCET and show that their effect in the factorization of Drell-Yan process is canceled[6]. Therefore our work completes the proof or argument of factorization of Drell-Yan processes in the framework of SCET.

Before our study of SCET by adding glauber gluons and the factorization of Drell-Yan processes, we show the existence of glauber gluons in scattering amplitudes. For this we consider the process  $q(p_1) + \bar{q}(p_2) \rightarrow g(k_g) + \gamma^*(q)$ , which contributes to the differential cross section of Drell-Yan

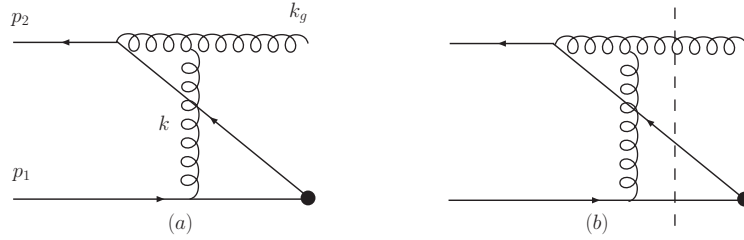


Figure 1: A diagram which contributes to the process  $q(p_1) + \bar{q}(p_2) \rightarrow g(k_g) + \gamma^*(q)$  at one loop level. The broken line is a cut. The black circle denotes the insertion of the electromagnetic current operator.

processes. At one loop level, the amplitude of the process receives a contribution from Fig.1a. We will use the light-cone coordinate system, in which a vector  $a^\mu$  is expressed as  $a^\mu = (a^+, a^-, \vec{a}_\perp) = ((a^0 + a^3)/\sqrt{2}, (a^0 - a^3)/\sqrt{2}, a^1, a^2)$  and  $a_\perp^2 = (a^1)^2 + (a^2)^2$ . We introduce two light cone vectors  $\bar{n}^\mu = (1, 0, 0, 0)$  and  $n^\mu = (0, 1, 0, 0)$ . The momenta are given as:

$$p_1^\mu = (p_1^+, 0, 0, 0), \quad p_2^\mu = (0, p_2^-, 0, 0), \quad k_g^\mu = (k_g^+, k_g^-, \vec{k}_{g\perp}). \quad (1)$$

We consider the contribution in the momentum configuration where the momentum  $k_g$  scales as  $k_g^\mu \sim Q(\lambda^2, 1, \lambda, \lambda)$  with  $Q$  as a large scale and  $\lambda \ll 1$ . In this configuration the outgoing gluon is collinear to the initial  $\bar{q}$ . For convenience we will take  $Q = 1$  in the following. The leading contribution from Fig.1 in the limit  $\lambda \rightarrow 0$  can come from different momentum regions of the loop momentum  $k$  with  $k^2 \sim \lambda^2$  or with  $k^2$  at higher order of  $\lambda$ . These possible regions are: The collinear regions with  $k$  at the order of  $(1, \lambda^2, \lambda, \lambda)$  or  $(\lambda^2, 1, \lambda, \lambda)$ , the soft region with  $k$  at the order of  $(\lambda, \lambda, \lambda, \lambda)$ , the ultra-soft region with  $k$  at the order of  $(\lambda^2, \lambda^2, \lambda^2, \lambda^2)$  and the glauber region with  $k$  at the order of  $(\lambda^2, \lambda^2, \lambda, \lambda)$ . Gluons with momenta in glauber regions are called as glauber gluons. To show that the glauber gluon gives nonzero contribution to the amplitude, it is enough to consider the following loop integral involved in the contribution of Fig.1a:

$$I = i \int \frac{d^4 k}{(2\pi)^4} \cdot \frac{1}{k^2 + i\varepsilon} \cdot \frac{1}{(p_1 - k)^2 + i\varepsilon} \cdot \frac{1}{(k_g - k)^2 + i\varepsilon} \cdot \frac{1}{(k - k_g + p_2)^2 + i\varepsilon}. \quad (2)$$

One can perform a power counting in  $\lambda$  for the integral in different momentum regions to determine the relative importance of contributions in different regions and leading contributions. One finds that the contributions from the collinear region with  $k \sim (\lambda^2, 1, \lambda, \lambda)$ , the ultra-soft region and the glauber region are at leading order of  $\lambda$  which is  $\mathcal{O}(\lambda^{-2})$ . The contributions from other regions are at higher order of  $\lambda$ . It is complicated to have a clear separation of leading contributions from the three regions, because they are all overlapped.

For our purpose we can consider the absorptive part of Fig.1a instead of the total contribution. We consider the cut diagram of Fig.1a, which is given as Fig.1b. Instead of the integral  $I$  we need to consider the absorptive- or imaginary part of  $I$ . The imaginary part is given by:

$$\text{Im} I = \frac{1}{2} \int \frac{d^4 k}{(2\pi)^4} \cdot \frac{1}{k^2 + i\varepsilon} \cdot (-2\pi i) \delta((p_1 - k)^2) \cdot \frac{1}{(k_g - k)^2 + i\varepsilon} (-2\pi i) \delta((k - k_g + p_2)^2). \quad (3)$$

From these on-shell conditions and with  $\vec{k}_\perp \sim (\lambda, \lambda)$  one always has  $k^+ \sim \lambda^2$  and  $k^- \sim \lambda^2$ . This indicates that the absorptive part of  $I$  at the leading order of  $\lambda$  is determined only by the glauber

region. Performing the power counting for the glauher region one obtains the leading contribution to  $\text{Im}I$  as:

$$\begin{aligned}\text{Im}I_G &= \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \cdot \frac{1}{-k_\perp^2 + i\varepsilon} \cdot (-2\pi i) \delta(-2p_1^+ k^- - k_\perp^2) \cdot \frac{1}{-2k_g^- k^+ - k_\perp^2 + i\varepsilon} \\ &\quad \cdot (-2\pi i) \delta(-2k^+ (k_g^- - p_2^-) + 2\vec{k}_{g\perp} \cdot \vec{k}_\perp - k_\perp^2 + (k_g - p_2)^2) \\ &= -\frac{1}{8p_1^+ p_2^-} \int \frac{d^2k_\perp}{(2\pi)^2} \frac{1}{k_\perp^2 (\vec{k}_\perp - \vec{k}_{g\perp})^2} = \frac{1}{8\pi s k_{g\perp}^2} \left[ \left( \frac{2}{\epsilon} - \gamma + \ln 4\pi \right) - \ln \frac{k_{g\perp}^2}{\mu^2} \right] + \mathcal{O}(\lambda^0),\end{aligned}\quad (4)$$

with  $s = 2p_1^+ p_2^-$ . In the above we have used dimensional regularization with  $\epsilon = 4 - d$  to regularize divergences. From the result the integral is divergent. The divergent contribution comes partly from the region of  $\vec{k}_\perp \sim 0$  and partly from the region of  $\vec{k}_\perp \sim \vec{k}_{g\perp}$ . The singular contribution from the region of  $\vec{k}_\perp \sim \vec{k}_{g\perp}$  is because that there are pinched poles in the complex  $k^+$ -plan for the integral  $I$ . The singularities generated by these pinched poles have brought up a difficulty to prove the factorization in Drell-Yan processes.

From the above discussion we have  $\text{Im}I = \text{Im}I_G(1 + \mathcal{O}(\lambda))$ . To verify this one can work out the exact result of the integral  $I$ . The exact result can be found in [8]. Through an analytical continuation and an expansion of the exact result we have from [8]:

$$\text{Im}I = \frac{1}{8\pi s k_{g\perp}^2} \left[ \left( \frac{2}{\epsilon} - \gamma + \ln 4\pi \right) - \ln \frac{k_{g\perp}^2}{\mu^2} \right] + \mathcal{O}(k_{g\perp}^0). \quad (5)$$

Comparing the result in Eq.(4,5) one verifies  $\text{Im}I = \text{Im}I_G(1 + \mathcal{O}(\lambda))$ . One can conclude that the exchange of glauher gluons does give nonzero contributions to scattering amplitudes at leading order of  $\lambda$  and hence possibly to differential cross sections. The possible contributions due to glauher gluons are at the same order of contributions due to collinear- and ultra-soft gluons. Therefore, one should include glauher gluons into SCET as an effective theory of QCD to describe interactions between collinear partons. Although the contributions of glauher gluons in Drell-Yan processes at twist-2 level are canceled, as shown in [2, 3] and in the current work, but the existence of glauher gluons has observable effects at twist-3 level. Recent studies of single transverse spin asymmetries in Drell-Yan- and DIS processes have been shown that these asymmetries are generated through exchanges of glauher gluons[9]. Hence it is also important to have SCET containing glauher gluons for a possible study of factorizations of single transverse spin asymmetries.

To derive SCET containing glauher gluons, we first consider SCET of one collinear system, later we will add another collinear system for our purpose. We will use the space-time representation of SCET as used in [7]. We choose the collinear system moving along the  $\bar{n}$ -direction with the momentum  $p$  at the order

$$p^\mu \sim Q(1, \lambda^2, \lambda, \lambda), \quad (6)$$

where  $Q$  is a larger scale while  $\lambda$  is small with  $\lambda \ll 1$ . Again we will take  $Q = 1$  in the following. Interacting particles in such a collinear system can be classified with their momenta at different orders of  $\lambda$ . They are collinear particles with momenta at the order  $p^\mu \sim (1, \lambda^2, \lambda, \lambda)$ , ultra-soft particles with momenta at the order  $p^\mu \sim (\lambda^2, \lambda^2, \lambda^2, \lambda^2)$ , and glauher gluons with momenta at the order  $p^\mu \sim (\lambda^2, \lambda^2, \lambda, \lambda)$ . The later have not been incorporated in SCET. Soft particles with momenta at the order of  $p^\mu \sim (\lambda, \lambda, \lambda, \lambda)$  will not interact with the above particles because of momentum conservation. However, their effect will appear in matching operators of full QCD into

those of SCET. It has been shown that their effect is canceled in Drell-Yan process. We will not consider soft particles here.

For the collinear system we decompose the gluon field and quark field of full QCD as:

$$A^\mu = A_{\bar{n}}^\mu + A_g^\mu + A_{us}^\mu, \quad \psi = \xi_{\bar{n}} + \eta + q. \quad (7)$$

In the above  $A_{\bar{n}}$  and  $\xi_{\bar{n}}$  are the collinear fields for collinear gluons and collinear quarks, respectively.  $\eta$  is a small component for collinear quarks which will be integrated out and expressed in terms of other fields.  $A_{us}$  and  $q$  are the ultra-soft fields for ultra-soft particles.  $A_g^\mu$  is the field for glauber gluons. The space-time dependence of these fields are characterized by the small scale  $\lambda$  in the following patten. For a collinear field  $\phi_{\bar{n}}$  which can be the collinear gluon field or the collinear quark field we have

$$\partial^+ \phi_{\bar{n}}(x) \sim \mathcal{O}(1) \phi_{\bar{n}}(x), \quad \partial^- \phi_{\bar{n}}(x) \sim \mathcal{O}(\lambda^2) \phi_{\bar{n}}(x), \quad \partial_\perp^\mu \phi_{\bar{n}}(x) \sim \mathcal{O}(\lambda) \phi_{\bar{n}}(x). \quad (8)$$

For a generic ultra-soft field  $\phi_{us}$  and the glauber gluon field we have

$$\begin{aligned} \partial^\mu \phi_{us}(x) &\sim \mathcal{O}(\lambda^2) \phi_{us}(x), \\ \partial^+ A_g^\mu(x) &\sim \mathcal{O}(\lambda^2) A_g^\mu(x), \quad \partial^- A_g^\mu(x) \sim \mathcal{O}(\lambda^2) A_g^\mu(x), \quad \partial_\perp^\nu A_g^\mu(x) \sim \mathcal{O}(\lambda) A_g^\mu(x). \end{aligned} \quad (9)$$

By inspecting propagators of particles in different momentum modes we can derive the following power-scaling:

$$\begin{aligned} A_{\bar{n}}^+ &\sim \mathcal{O}(1), \quad A_{\bar{n}}^- \sim \mathcal{O}(\lambda^2), \quad A_{\bar{n}\perp}^\mu \sim \mathcal{O}(\lambda), \quad \xi \sim \mathcal{O}(\lambda), \\ A_{us}^\mu &\sim \mathcal{O}(\lambda^2), \quad A_g^\mu \sim \mathcal{O}(\lambda^2), \quad q \sim \mathcal{O}(\lambda^3). \end{aligned} \quad (10)$$

Using the power-scaling of fields and the scaling of the space-time dependence of the fields one can expand the Lagrangian of full QCD in  $\lambda$  with the above fields. The expansion is straightforward. To express the leading order results we introduce the modified derivative  $\partial_{\bar{n}}^\mu$  or projection. The derivative  $\partial_{\bar{n}}^+$  only acts on collinear fields, it gives zero if the derivative acts on ultra-soft- or glauber gluon fields, i.e.,

$$\partial_{\bar{n}}^+ \phi_{\bar{n}}(x) = \partial^+ \phi_{\bar{n}}(x), \quad \partial_{\bar{n}}^+ \phi_{us}(x) = 0, \quad \partial_{\bar{n}}^+ A_g^\mu = 0. \quad (11)$$

The derivative  $\partial_{\bar{n}}^-$  acts as an usual derivative on all fields while  $\partial_{\bar{n}\perp}^\mu$  acts only on collinear- or glauber gluon fields, but not on ultra-soft fields. The leading terms of our effective theory can be written as:

$$\mathcal{L}^{(\bar{n})}(\xi, q, A_{\bar{n}}^\mu, A_g^\mu, A_{us}^\mu) = \mathcal{L}_g(A_g^\mu) + \mathcal{L}_{us}(q, A_{us}^\mu) + \mathcal{L}_{\bar{n}}(\xi_{\bar{n}}, A_{\bar{n}}^\mu, \bar{n} \cdot A_g, \bar{n} \cdot A_{us}) + \dots, \quad (12)$$

where  $\dots$  stand for terms whose contributions to the action  $S = \int d^4x \mathcal{L}^{(\bar{n})}$  are suppressed by  $\lambda$ . In the expansion of the action  $S$  one should assign the space-time integration  $d^4x$  with a correct scaling in  $\lambda$  by noticing that the integration should not change the momentum orders of fields in different momentum modes.

At leading order glauber gluons can not interact with ultra-soft particles and they also can not interact with themselves. Hence  $\mathcal{L}_g(A_g)$  is only the kinetic term of the glauber gluons:

$$\mathcal{L}_g = -\frac{1}{2} \text{Tr}[\partial_\perp^\mu A_g^\nu - \partial_\perp^\nu A_g^\mu][\partial_{\perp\mu} A_{g\nu} - \partial_{\perp\nu} A_{g\mu}]. \quad (13)$$

It should be noted that only the transverse derivative appears. This fact results in that the propagator of glauher gluons with the momentum  $k$  will be proportional to  $1/k_\perp^2$ , as already seen in Eq.(4).  $\mathcal{L}_{us}(q, A_{us}^\mu)$  is the part for ultra-soft particles:

$$\mathcal{L}_{us}(q, A_{us}^\mu) = \bar{q} i \gamma \cdot D_{us} q - \frac{1}{2g^2} \text{Tr} \{ [D_{us}^\mu, D_{us}^\nu] \}^2, \quad (14)$$

where the covariant derivative is defined only with the ultra-soft gluon field, i.e.,  $D_{us}^\mu = \partial^\mu + ig A_{us}^\mu$ .  $\mathcal{L}_{\bar{n}}(\xi_{\bar{n}}, A_{\bar{n}}, A_g, A_{us})$  contains collinear particles and their interactions with ultra-soft- and glauher gluons. To express it we introduce the covariant derivative with the modified derivative:

$$D_{\bar{n}}^\mu = \partial_{\bar{n}}^\mu + ig A_{\bar{n}}^\mu + ig n^\mu \bar{n} \cdot (A_g + A_{us}). \quad (15)$$

With the covariant derivative the part  $\mathcal{L}_{\bar{n}}$  can be expressed as

$$\begin{aligned} \mathcal{L}_{\bar{n}}(\xi_{\bar{n}}, A_{\bar{n}}^\mu, \bar{n} \cdot A_g, \bar{n} \cdot A_{us}) &= \bar{\xi}_{\bar{n}} i n \cdot \gamma \bar{n} \cdot D_{\bar{n}} \xi_{\bar{n}} - \bar{\xi}_{\bar{n}} i \gamma \cdot D_{c\perp} \frac{1}{2iD_c^+} n \cdot \gamma i \gamma \cdot D_{c\perp} \xi_{\bar{n}} \\ &\quad - \frac{1}{2g^2} \text{Tr} \{ [D_{\bar{n}}^\mu, D_{\bar{n}}^\nu] \}^2, \end{aligned} \quad (16)$$

where the collinear covariant derivative  $D_c$  is defined as  $D_c^\mu = \partial^\mu + ig A_{\bar{n}}^\mu$ . The glauher gluon field appears in the covariant derivative in the same way as the ultra-soft gluon field. However, the interaction of collinear gluons with ultra-soft gluons are different than that with glauher gluons because the operator  $\partial_{\bar{n}\perp}$  does not act on the ultra-soft field, but on the glauher field, i.e.,  $\partial_{\bar{n}\perp}^\mu A_g \neq 0$ .

The effective theory at leading order  $\lambda$  is invariant under various gauge transformations, which are collinear-, ultra-soft- and glauher gauge transformation. The space-time dependence of these transformations is characterized in the same patten as that of the corresponding fields in Eq. (8,9). These transformations are:

$$\begin{aligned} \text{Collinear : } A_{us}^\mu &\rightarrow A_{us}^\mu, \quad A_g^\mu \rightarrow A_g^\mu, \quad \xi_{\bar{n}} \rightarrow U_c \xi_{\bar{n}}, \quad q \rightarrow q, \\ A_{\bar{n}}^\mu &\rightarrow U_c A_{\bar{n}}^\mu U_c^\dagger - \frac{i}{g} U_c \left[ \partial^\mu + ig(A_{us}^\mu + A_g^\mu), U_c^\dagger \right], \\ \text{Ultra - soft : } A_{\bar{n}}^\mu &\rightarrow U_{us} A_{\bar{n}}^\mu U_{us}^\dagger, \quad A_g^\mu \rightarrow U_{us} A_g^\mu U_{us}^\dagger, \quad \xi_{\bar{n}} \rightarrow U_{us} \xi_{\bar{n}}, \quad q \rightarrow U_{us} q, \\ A_{us}^\mu &\rightarrow U_{us} A_{us}^\mu U_{us}^\dagger - \frac{i}{g} U_{us} \left[ \partial^\mu, U_{us}^\dagger \right], \\ \text{Glauber : } A_g^\mu &\rightarrow A_g^\mu, \quad A_{us}^\mu \rightarrow A_{us}^\mu, \quad \xi_{\bar{n}} \rightarrow U_g \xi_{\bar{n}}, \quad q \rightarrow q, \\ A_{\bar{n}}^\mu &\rightarrow U_g A_{\bar{n}}^\mu U_g^\dagger - \frac{i}{g} U_g \left[ \partial^\mu + ig(A_{us}^\mu + A_g^\mu), U_g^\dagger \right]. \end{aligned} \quad (17)$$

Under these transformations the effective Lagrangian is invariant at the leading order of  $\lambda$ , or the change of the effective Lagrangian is suppressed by  $\lambda$ . One can re-express these transformations with the modified derivative and neglect higher order effects introduced by the fields in the transformation so that the effective Lagrangian is exactly invariant. In the above glauher gauge transformation the glauher field is not changed. However, there is another type of transformations related to the glauher transformation defined by  $U_{g\perp}(\tilde{x}) = U_g(x^+ \bar{n} + x^- n)$ , under which the glauher field is changed:

$$\begin{aligned} A_g^\mu &\rightarrow U_{g\perp} A_g^\mu U_{g\perp}^\dagger - \frac{i}{g} U_{g\perp} \left[ \partial^\mu, U_{g\perp}^\dagger \right], \quad A_{us}^\mu \rightarrow A_{us}^\mu, \quad q \rightarrow q, \\ A_{\bar{n}}^\mu &\rightarrow U_{g\perp} A_{\bar{n}}^\mu U_{g\perp}^\dagger + U_{g\perp} \left[ A_{us}^\mu, U_{g\perp}^\dagger \right], \quad \xi_{\bar{n}} \rightarrow U_{g\perp} \xi_{\bar{n}}. \end{aligned} \quad (18)$$

Under the above transformation the Lagrangian is invariant. One needs to fix gauges for different gauge fields and introduces corresponding ghost fields. We note that the gauge fixing for ultra-soft- and glauher gluons can be fixed separately. We will take Feynman gauge for ultra-soft- and glauher gluons. For collinear gluons we take the covariant gauge for collinear gluons as in [5], where the covariant derivative should include the glauher gluon field as in Eq.(15).

To study the effects of glauher- and ultra-soft gluons in Drell-Yan process one needs to include another collinear fields in the  $n$ -direction. This can be done by adding the collinear gluon field  $A_n^\mu$  and the collinear quark field  $\xi_n$  in the decomposition in Eq.(7) correspondingly and by doing the expansion in  $\lambda$ . At leading order  $\lambda$  the collinear fields in different directions can no interact with each other. Momentum conservation insures that the collinear fields in different directions can not be mixed with each other. Then, the soft collinear effective theory for two collinear systems moving in the  $n$ -direction and in  $\bar{n}$  direction respectively can be easily written down:

$$\mathcal{L}^{(\bar{n}n)} = \mathcal{L}_g(A_g^\mu) + \mathcal{L}_{us}(q, A_{us}^\mu) + \mathcal{L}_{\bar{n}}(\xi_{\bar{n}}, A_{\bar{n}}^\mu, \bar{n} \cdot A_g, \bar{n} \cdot A_{us}) + \mathcal{L}_n(\xi_n, A_n^\mu, n \cdot A_g, n \cdot A_{us}) + \dots \quad (19)$$

From the effective theory, the interaction between the collinear fields in different directions is only through exchanging glauher- and ultra-soft gluons.

Without the presence of glauher gluons the decoupling of ultra-soft gluons can be simply made through redefinitions of collinear fields, as shown in [5, 6]. In the presence of glauher gluons in SCET, one can also redefine fields so that the redefined fields do not interacting with glauher gluons and ultra-soft gluons. We introduce the following  $SU(3)$ -matrices:

$$\begin{aligned} Z_{\bar{n}}(x) &= P \exp \left\{ -ig \int_{-\infty}^0 d\beta \bar{n} \cdot A_g(\beta \bar{n} + x) \right\}, \quad Z_n(x) = P \exp \left\{ -ig \int_{-\infty}^0 d\beta n \cdot A_g(\beta n + x) \right\}, \\ Y_{\bar{n}}(x) &= P \exp \left\{ -ig \int_{-\infty}^0 d\beta \bar{n} \cdot A_{us}(\beta \bar{n} + x) \right\}, \quad Y_n(x) = P \exp \left\{ -ig \int_{-\infty}^0 d\beta n \cdot A_{us}(\beta n + x) \right\}. \end{aligned} \quad (20)$$

With these matrices we redefine all collinear fields as:

$$\begin{aligned} A_{\bar{n}}^\mu &= Z_{\bar{n}} \left[ Y_{\bar{n}} \tilde{A}_{\bar{n}}^\mu Y_{\bar{n}}^\dagger \right] Z_{\bar{n}}^\dagger - \frac{i}{g} Z_{\bar{n}} \left[ \partial_\perp^\mu + ig_s n^\mu A_{us}^-, Z_{\bar{n}}^\dagger \right], \quad \xi_{\bar{n}} = Z_{\bar{n}} Y_{\bar{n}} \chi_{\bar{n}}, \\ A_n^\mu &= Z_n \left[ Y_n \tilde{A}_n^\mu Y_n^\dagger \right] Z_n^\dagger - \frac{i}{g} Z_n \left[ \partial_\perp^\mu + ig_s \bar{n}^\mu A_{us}^+, Z_n^\dagger \right], \quad \xi_n = Z_n Y_n \chi_n. \end{aligned} \quad (21)$$

With the re-defined fields the Lagrangian containing the interactions with glauher gluons at the leading order of  $\lambda$  becomes

$$\mathcal{L}_{\bar{n}}(\xi_{\bar{n}}, A_{\bar{n}}^\mu, \bar{n} \cdot A_g, \bar{n} \cdot A_{us}) = \mathcal{L}_{\bar{n}}(\chi_{\bar{n}}, \tilde{A}_{\bar{n}}^\mu, 0, 0), \quad \mathcal{L}_n(\xi_n, A_n^\mu, n \cdot A_g, n \cdot A_{us}) = \mathcal{L}_n(\chi_n, \tilde{A}_n^\mu, 0, 0). \quad (22)$$

This result tells that the redefined collinear fields will not interact with glauher gluons and ultra-soft gluons. It should be noted that there is no difference between  $S$ -matrix elements calculated with the original Lagrangian and those calculated with the Lagrangian of redefined fields. For our redefinition of fields this can also be realized with path-integral of SCET by noting the fact that the Jacobian's of changing field variables are unit.

Now we are in position to discuss the factorization for Drell-Yan process. The process is given as

$$h_{\bar{n}} + h_n \rightarrow \ell^+ + \ell^- + X, \quad (23)$$

where one hadron  $h_n$  moves in the  $n$ -direction while another  $h_{\bar{n}}$  moves in the  $\bar{n}$ -direction. At leading order of QED the differential cross-section is determined by the hadronic tensor:

$$W^{\mu\nu}(q) = \int d^4x e^{-iq \cdot x} \langle h_{\bar{n}} h_n | J^\mu(x) J^\nu(0) | h_n h_{\bar{n}} \rangle, \quad J^\mu = \bar{\psi} \gamma^\mu \psi. \quad (24)$$

To show the factorization one needs first to match the operator  $J^\mu(x) J^\nu(0)$  to the operators of the soft collinear effective theory at the leading order in  $\lambda$ . In the matching, the interactions between collinear particles which are not presented in SCET are assembled by different Wilson lines:

$$W_{\bar{n}}(x) = P \exp \left\{ -ig \int_{-\infty}^0 d\alpha n \cdot A_{\bar{n}}(\alpha n + x) \right\}, \quad W_n(x) = P \exp \left\{ -ig \int_{-\infty}^0 d\alpha \bar{n} \cdot A_n(\alpha \bar{n} + x) \right\}. \quad (25)$$

At leading order of  $\lambda$  the operators consists of collinear fields only and their relative space-time dependence can only be of  $x^- n$  for  $\phi_{\bar{n}}$  fields and of  $x^+ \bar{n}$  of  $\phi_n$ . Taking the operators consisting only of quark fields in SCET as example, the matching is:

$$W^{\mu\nu}(q) = \int dk_1^+ dk_2^- C_{ij}^{\mu\nu}(q, k_1^+ \bar{n}, k_2^- n) \int dx^- dy^+ \exp \left( -ix^- k_1^+ - iy^+ k_2^- \right) \cdot \langle h_{\bar{n}} h_n | \left[ (\bar{\xi}_{\bar{n}} W_{\bar{n}})(x^- n) \Gamma_{\bar{n}}^{(i)}(W_{\bar{n}}^\dagger \xi_{\bar{n}})(0) \right] \left[ (\bar{\xi}_n W_n)(y^+ \bar{n}) \Gamma_n^{(j)}(W_n^\dagger \xi_n)(0) \right] | h_n h_{\bar{n}} \rangle + \dots \quad (26)$$

where  $\dots$  stand for other possible operators at leading order and terms at higher orders in  $\lambda$  which will be neglected. The leading order is at  $\lambda^4$ . For simplicity we will discuss the factorization for the above term in detail. Those terms containing other possible operators can be handled in the same way. In the above we have already used the properties

$$\bar{n} \cdot \gamma \xi_{\bar{n}} = \gamma^- \xi_{\bar{n}} = 0, \quad n \cdot \gamma \xi_n = \gamma^+ \xi_n = 0 \quad (27)$$

to decouple the Dirac indices. Hence the  $\Gamma_{\bar{n}}$ -matrices are given by

$$\left( \Gamma_{\bar{n}}^{(1)}, \Gamma_{\bar{n}}^{(2)}, \Gamma_{\bar{n}}^{(3)} \right) = (n \cdot \gamma, n \cdot \gamma \gamma_5, n \cdot \gamma \gamma_\perp^\nu). \quad (28)$$

$\Gamma_n^{(i)}$  is defined by replacing  $n$  with  $\bar{n}$  in  $\Gamma_{\bar{n}}^{(i)}$ .

The functions  $C$  can be calculated with perturbative theory. If one uses perturbative theory to calculate the hadronic tensor by replacing the hadrons with parton states, one will have I.R.- and collinear singularities. These singularities are reproduced by the matrix elements of SCET operators through construction of SCET, e.g., those in the right hand side of the above equation. Hence the perturbative functions are free from I.R.- and collinear divergence. At this stage the long-distance- and short-distance effects are factorized, but the factorization of Drell-Yan process is not complete. The factorization is completely achieved if one can show that the matrix element of the two-hadron state in Eq.(26) can be factorized into a product of two matrix elements of one-hadron state. In Eq.(26) the operator  $\left[ (\bar{\xi}_{\bar{n}} W_{\bar{n}})(x^- n) \Gamma_{\bar{n}}^{(i)}(W_{\bar{n}}^\dagger \xi_{\bar{n}})(0) \right]$  can emit ultra-soft- and glauher gluons to interact with the state  $h_n$  and the emitted gluons can also be absorbed by the operator  $\left[ (\bar{\xi}_n W_n)(y^+ \bar{n}) \Gamma_n^{(j)}(W_n^\dagger \xi_n)(0) \right]$ . This prevents us at first look from factorizing the matrix element of two-hadron state in Eq.(26) as a product of two matrix elements of one-hadron state. For the factorization one needs to show that the total effects of gluon-emissions are canceled.

Before discussing the factorization, we discuss a general feature of the  $\lambda$ -expansion in the space-time representation. We consider a product of collinear fields  $\phi_{\bar{n}}$ , denoted as  $\Phi_{\bar{n}}$ , multiplied with any ultra-soft field  $\phi_{us}$ . The integral of the product has the following property:

$$\int d\beta \Phi_{\bar{n}}(\beta n + x) \phi_{us}(\beta n + x) = \left[ \int d\beta \Phi_{\bar{n}}(\beta n + x) \right] \phi_{us}(x) \left\{ 1 + \mathcal{O}(\lambda^2) \right\}. \quad (29)$$

This equation can be derived by expansion the ultra-soft field as  $\phi_{us}(\beta n + x) = \phi_{us}(x) + \beta \partial^+ \phi_{us}(x) + \dots$ . The derivative  $\partial^+ \phi_{us}(x)$  is at the order of  $\lambda^2$ , while the integration variable  $\beta$  is related to the collinear direction and is taken as at the order of  $\lambda^0$ . Therefore we have the above equation. Now we use the redefined collinear fields in Eq.(21) to express the matrix elements in Eq.(26). Under the re-definition the Wilson  $W_n$  and  $W_{\bar{n}}$  become by using the property

$$\begin{aligned} W_n(x) &= V_n(x) \tilde{W}_n(x) V_n^\dagger(x) \left\{ 1 + \mathcal{O}(\lambda^2) \right\}, & V_n(x) &= Z_n(x) Y_n(x), \\ W_{\bar{n}}(x) &= V_{\bar{n}}(x) \tilde{W}_{\bar{n}}(x) V_{\bar{n}}^\dagger(x) \left\{ 1 + \mathcal{O}(\lambda^2) \right\}, & V_{\bar{n}}(x) &= Z_{\bar{n}}(x) Y_{\bar{n}}(x) \end{aligned} \quad (30)$$

where  $\tilde{W}_n$  and  $\tilde{W}_{\bar{n}}$  are defined by replacing  $A_n$  and  $A_{\bar{n}}$  in Eq.(25) with  $\tilde{A}_n$  and  $\tilde{A}_{\bar{n}}$ , respectively. The hadronic matrix element with the redefined fields become:

$$\begin{aligned} &\langle h_{\bar{n}} h_n | \left[ (\bar{\xi}_{\bar{n}} W_{\bar{n}})(x^- n) \Gamma_{\bar{n}}^{(i)}(W_{\bar{n}}^\dagger \xi_{\bar{n}})(0) \right] \left[ (\bar{\xi}_n W_n)(y^+ \bar{n}) \Gamma_n^{(j)}(W_n^\dagger \xi_n)(0) \right] | h_n h_{\bar{n}} \rangle \\ &= \langle h_{\bar{n}} h_n | (\bar{\chi}_{\bar{n}} \tilde{W}_{\bar{n}})_i(x^- n) \Gamma_{\bar{n}}^{(i)}(\tilde{W}_{\bar{n}}^\dagger \chi_{\bar{n}})_j(0) \left[ V_{\bar{n}}^\dagger(x^- n) V_{\bar{n}}(0) \right]_{ij} \left[ V_n^\dagger(y^+ \bar{n}) V_n(0) \right]_{kl} \\ &\quad (\bar{\chi}_n \tilde{W}_n)_k(y^+ \bar{n}) \Gamma_n^{(j)}(\tilde{W}_n^\dagger \chi_n)_l(0) | h_n h_{\bar{n}} \rangle \left[ 1 + \mathcal{O}(\lambda^2) \right], \end{aligned} \quad (31)$$

where  $ijkl$  are color indices. With the definition of  $V_{\bar{n}}(x^- n)$  and  $V_n(y^+ \bar{n})$  one knows that the  $x^-$ -dependence in  $V_{\bar{n}}(x^- n)$  and the  $y^+$ -dependence in  $V_n(y^+ \bar{n})$  are characterized by  $\lambda^2$ . Since the space-time separation  $x^-$  and  $y^+$  will be integrated over in the hadronic tensor, hence we can have the following approximation under the integrations:

$$\left[ V_{\bar{n}}^\dagger(x^- n) V_{\bar{n}}(0) \right]_{ij} \left[ V_n^\dagger(y^+ \bar{n}) V_n(0) \right]_{kl} = \delta_{ij} \delta_{lk} + \mathcal{O}(\lambda^2). \quad (32)$$

Using the fact that redefined collinear fields do not interact with glauher- and ultra-soft gluons and also that the collinear fields in different directions do not interact, we can re-write Eq.(31) under the integrations in Eq.(26) with Eq.(32) as:

$$\begin{aligned} &\langle h_{\bar{n}} h_n | \left[ (\bar{\xi}_{\bar{n}} W_{\bar{n}})(x^- n) \Gamma_{\bar{n}}^{(i)}(W_{\bar{n}}^\dagger \xi_{\bar{n}})(0) \right] \left[ (\bar{\xi}_n W_n)(y^+ \bar{n}) \Gamma_n^{(j)}(W_n^\dagger \xi_n)(0) \right] | h_n h_{\bar{n}} \rangle \\ &= \left[ \langle h_{\bar{n}} | (\bar{\chi}_{\bar{n}} \tilde{W}_{\bar{n}})(x^- n) \Gamma_{\bar{n}}^{(i)}(\tilde{W}_{\bar{n}}^\dagger \chi_{\bar{n}})(0) | h_{\bar{n}} \rangle \right] \left[ \langle h_n | (\bar{\chi}_n \tilde{W}_n)(y^+ \bar{n}) \Gamma_n^{(j)}(\tilde{W}_n^\dagger \chi_n)(0) | h_n \rangle \right] \\ &\quad \cdot \left\{ 1 + \mathcal{O}(\lambda^2) \right\}. \end{aligned} \quad (33)$$

Substituting this result and expressing the redefined fields with the original fields, we complete the factorization for Eq. (26):

$$\begin{aligned} W^{\mu\nu}(q) &= \int dk_1^+ dk_2^- C_{ij}^{\mu\nu}(q, k_1^+ \bar{n}, k_2^- n) \cdot \left[ \int dx^- e^{-ix^- k_1^+} \langle h_{\bar{n}} | (\bar{\xi}_{\bar{n}} W_{\bar{n}})(x^- n) \Gamma_{\bar{n}}^{(i)}(W_{\bar{n}}^\dagger \xi_{\bar{n}})(0) | h_{\bar{n}} \rangle \right] \\ &\quad \cdot \left[ \int dy^+ e^{-iy^+ k_2^-} \langle h_n | (\bar{\xi}_n W_n)(y^+ \bar{n}) \Gamma_n^{(j)}(W_n^\dagger \xi_n)(0) | h_n \rangle \right] \left[ 1 + \mathcal{O}(\lambda^2) \right] + \dots \end{aligned} \quad (34)$$



In the matching in Eq.(26) there are contributions from color-octet operators at leading order like

$$\left[ (\bar{\xi}_{\bar{n}} W_{\bar{n}})(x^- n) \Gamma_n^{(i)} T^a (W_n^\dagger \xi_n)(0) \right] \left[ (\bar{\xi}_n W_n)(y^+ \bar{n}) \Gamma_n^{(j)} T^a (W_n^\dagger \xi_n)(0) \right], \quad (35)$$

represented by  $\dots$ . With the above procedure and the fact that the hadrons are color-less, one can show that these color-octet operators do not contribute at leading order of  $\lambda$ .

In the above we have shown that the effects of glauher gluons are completely canceled at the leading order of  $\lambda$  in the same way of the cancelation of ultra-soft gluons. The above result is invariant at leading order of  $\lambda$  under various gauge transformation. We note that the Wilson line  $W_n$  and  $W_{\bar{n}}$  under the transformations given in Eq. (17) are transformed as:

$$\begin{aligned} \text{Collinear : } W_n &\rightarrow U_n W_n \left(1 + \mathcal{O}(\lambda^2)\right), & W_{\bar{n}} &\rightarrow U_{\bar{n}} W_{\bar{n}} \left(1 + \mathcal{O}(\lambda^2)\right). \\ \text{Ultra - soft : } W_n &\rightarrow U_{us} W_n U_{us}^\dagger \left(1 + \mathcal{O}(\lambda^2)\right), & W_{\bar{n}} &\rightarrow U_{us} W_{\bar{n}} U_{us}^\dagger \left(1 + \mathcal{O}(\lambda^2)\right). \\ \text{Glauber : } W_n &\rightarrow U_g W_n U_g^\dagger \left(1 + \mathcal{O}(\lambda^2)\right), & W_{\bar{n}} &\rightarrow U_g W_{\bar{n}} U_g^\dagger \left(1 + \mathcal{O}(\lambda^2)\right). \end{aligned} \quad (36)$$

At the leading order of  $\lambda$  there are other three operators appearing in the matching in Eq.(26). They are:

$$\bar{\xi}_n \xi_n G_n^{+\mu} G_n^{+\nu}, \quad \bar{\xi}_{\bar{n}} \xi_{\bar{n}} G_n^{-\mu} G_n^{-\nu}, \quad G_n^{-\mu} G_n^{-\nu} G_n^{+\mu'} G_n^{+\nu'}, \quad (37)$$

where we have suppressed the color structure, spin structures, space-time dependence and the Wilson line of collinear gluon in different directions. The collinear field strength tensors are defined with corresponding collinear gluon fields, the indices  $\mu, \nu, \mu'$  and  $\nu'$  are transverse. The factorization or the cancelation of effects through exchanges of glauher- and ultra-soft gluons can be shown in a similar way as in the above. With the factorization one can define for each hadron various parton distributions and write the final result for the hadronic tensor in a compact form. The detailed results can be found somewhere in the literature.

To summarize: In this letter we have studied the effects of glauher gluons in the framework of SCET, which had not been considered in the originally proposed SCET. We have confirmed the existence of glauher gluons through an example. By adding the glauher gluons into SCET we have found that the glauher gluons only interact with collinear particles. Furthermore, we have shown in the framework of SCET with glauher gluons that the effects of glauher gluons are canceled in Drell-Yan process, i.e., the factorization in the present of glauher gluons still holds. Therefore our work completes the proof or argument of factorization of Drell-Yan process in the framework of SCET. Our result is in the agreement with the traditional proof of Drell-Yan process where the existence of glauher gluon has been a challenging difficulty. However the proof here is given in a more transparent way.

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